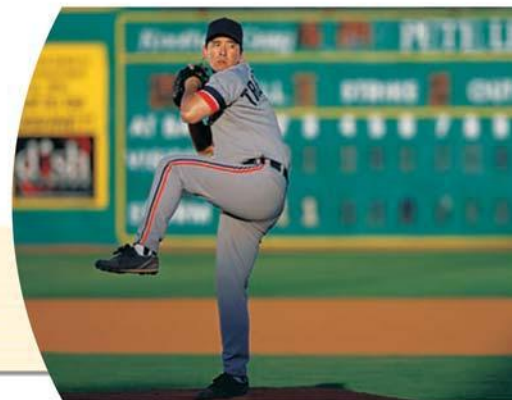# 2.1 Use Inductive Reasoning

Before	You classified polygons by the number of sides.
Now	You will describe patterns and use inductive reasoning.
Why?	So you can make predictions about baseball, as in Ex. 32.

Key Vocabulary
• conjecture
• inductive reasoning
• counterexample

Geometry, like much of science and mathematics, was developed partly as a result of people recognizing and describing patterns. In this lesson, you will discover patterns yourself and use them to make predictions.

EXAMPLE 1 Describe a visual pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.
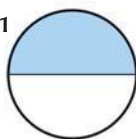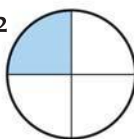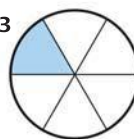
Figure 1   Figure 2   Figure 3

Solution

Each circle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing a circle into eighths. Shade the section just above the horizontal segment at the left.
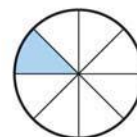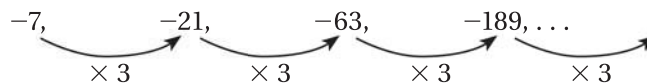
Figure 4

EXAMPLE 2 Describe a number pattern

Describe the pattern in the numbers −7, −21, −63, −189, . . . and write the next three numbers in the pattern.

Notice that each number in the pattern is three times the previous number.

$$-7, \quad -21, \quad -63, \quad -189, \ldots$$
$$\times 3 \quad \times 3 \quad \times 3 \quad \times 3$$

▶ Continue the pattern. The next three numbers are −567, −1701, and −5103.

Animated **Geometry** at classzone.com

✓ **GUIDED PRACTICE** for Examples 1 and 2

1. Sketch the fifth figure in the pattern in Example 1.

2. *Describe* the pattern in the numbers 5.01, 5.03, 5.05, 5.07, ... Write the next three numbers in the pattern.

INDUCTIVE REASONING A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

EXAMPLE 3 Make a conjecture

Given five collinear points, make a conjecture about the number of ways to connect different pairs of the points.

Solution

Make a table and look for a pattern. Notice the pattern in how the number of connections increases. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture	•	•—•	⌒•—•—•	•⌒•—•—•⌒	•⌒•—•—•—•⌒
Number of connections	0	1	3	6	?

+ 1   + 2   + 3   + **?**

▶ **Conjecture** You can connect five collinear points 6 + **4**, or 10 different ways.

EXAMPLE 4 Make and test a conjecture

Numbers such as 3, 4, and 5 are called *consecutive numbers*. Make and test a conjecture about the sum of any three consecutive numbers.

Solution

STEP 1 **Find** a pattern using a few groups of small numbers.

$$3 + 4 + 5 = 12 = 4 \cdot 3 \qquad 7 + 8 + 9 = 24 = 8 \cdot 3$$

$$10 + 11 + 12 = 33 = 11 \cdot 3 \qquad 16 + 17 + 18 = 51 = 17 \cdot 3$$

▶ **Conjecture** The sum of any three consecutive integers is three times the second number.

STEP 2 **Test** your conjecture using other numbers. For example, test that it works with the groups −1, 0, 1 and 100, 101, 102.

$$-1 + 0 + 1 = 0 = 0 \cdot 3 \checkmark \qquad 100 + 101 + 102 = 303 = 101 \cdot 3 \checkmark$$

✓ **GUIDED PRACTICE** for Examples 3 and 4

3. Suppose you are given seven collinear points. Make a conjecture about the number of ways to connect different pairs of the points.

4. Make and test a conjecture about the sign of the product of any three negative integers.

DISPROVING CONJECTURES To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by simply finding one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

EXAMPLE 5 **Find a counterexample**

A student makes the following conjecture about the sum of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The sum of two numbers is always greater than the larger number.

Solution

To find a counterexample, you need to find a sum that is less than the larger number.

$$-2 + -3 = -5$$
$$-5 \not> -3$$

▸ Because a counterexample exists, the conjecture is false.

★ **EXAMPLE 6** **Standardized Test Practice**

Which conjecture could a high school athletic director make based on the graph at the right?

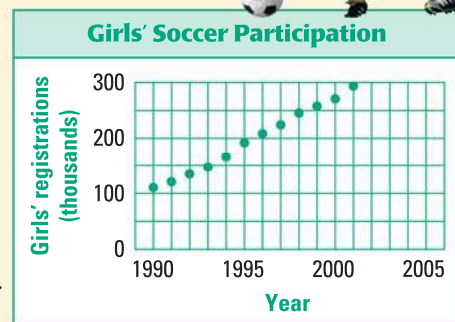**ELIMINATE CHOICES**
Because the graph does not show data about boys or the World Cup games, you can eliminate choices A and C.

(A) More boys play soccer than girls.

(B) More girls are playing soccer today than in 1995.

(C) More people are playing soccer today than in the past because the 1994 World Cup games were held in the United States.

(D) The number of girls playing soccer was more in 1995 than in 2001.

Girls' Soccer Participation

Solution

Choices A and C can be eliminated because they refer to facts not presented by the graph. Choice B is a reasonable conjecture because the graph shows an increase from 1990–2001, but does not give any reasons for that increase.

▸ The correct answer is B. (A) (B) (C) (D)

✓ **GUIDED PRACTICE** for Examples 5 and 6

5. Find a counterexample to show that the following conjecture is false.

   Conjecture The value of x^2 is always greater than the value of x.

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

2.1 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Write a definition of *conjecture* in your own words.

2. ★ **WRITING** The word *counter* has several meanings. Look up the word in a dictionary. Identify which meaning helps you understand the definition of *counterexample*.

EXAMPLE 1
on p. 72
for Exs. 3–5

SKETCHING VISUAL PATTERNS Sketch the next figure in the pattern.

3.   4.

5. ★ **MULTIPLE CHOICE** What is the next figure in the pattern?

(A) (B) (C) (D)

EXAMPLE 2
on p. 72
for Exs. 6–11

DESCRIBING NUMBER PATTERNS *Describe* the pattern in the numbers. Write the next number in the pattern.

6. 1, 5, 9, 13, . . .   7. 3, 12, 48, 192, . . .   8. 10, 5, 2.5, 1.25, . . .

9. 4, 3, 1, −2, . . .   10. $1, \frac{2}{3}, \frac{1}{3}, 0, \ldots$ 11. −5, −2, 4, 13, . . .

MAKING CONJECTURES In Exercises 12 and 13, copy and complete the conjecture based on the pattern you observe in the specific cases.

EXAMPLE 3
on p. 73
for Ex. 12

12. Given seven noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Number of points	3	4	5	6	7
Picture					?
Number of connections	3	6	10	15	?

Conjecture You can connect seven noncollinear points __?__ different ways.

EXAMPLE 4
on p. 73
for Ex. 13

13. Use these sums of odd integers: 3 + 7 = 10, 1 + 7 = 8, 17 + 21 = 38

Conjecture The sum of any two odd integers is __?__.
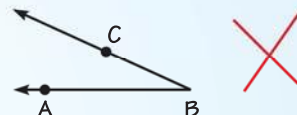
EXAMPLE 5
on p. 74
for Exs. 14–17

FINDING COUNTEREXAMPLES In Exercises 14–17, show the conjecture is false by finding a counterexample.

14. If the product of two numbers is positive, then the two numbers must both be positive.

15. The product $(a + b)^2$ is equal to $a^2 + b^2$, for $a \neq 0$ and $b \neq 0$.

16. All prime numbers are odd.

17. If the product of two numbers is even, then the two numbers must both be even.

18. ERROR ANALYSIS *Describe* and correct the error in the student's reasoning.

True conjecture: All angles are acute.

Example:

19. ★ **SHORT RESPONSE** *Explain* why only one counterexample is necessary to show that a conjecture is false.

xy ALGEBRA In Exercises 20 and 21, write a function rule relating x and y.

20.

x	1	2	3
y	−3	−2	−1

21.

x	1	2	3
y	2	4	6

22. ★ **MULTIPLE CHOICE** What is the first number in the pattern?

$$\underline{\ ?\ }, \underline{\ ?\ }, \underline{\ ?\ }, 81, 243, 729$$

(A) 1 **(B)** 3 **(C)** 9 **(D)** 27

MAKING PREDICTIONS *Describe* a pattern in the numbers. Write the next number in the pattern. Graph the pattern on a number line.

23. $2, \dfrac{3}{2}, \dfrac{4}{3}, \dfrac{5}{4}, \ldots$

24. $1, 8, 27, 64, 125, \ldots$

25. $0.45, 0.7, 0.95, 1.2, \ldots$

26. $1, 3, 6, 10, 15, \ldots$

27. $2, 20, 10, 100, 50, \ldots$

28. $0.4(6), 0.4(6)^2, 0.4(6)^3, \ldots$

29. **xy ALGEBRA** Consider the pattern $5, 5r, 5r^2, 5r^3, \ldots$. For what values of r will the values of the numbers in the pattern be increasing? For what values of r will the values of the numbers be decreasing? *Explain*.

30. REASONING A student claims that the next number in the pattern $1, 2, 4, \ldots$ is 8, because each number shown is two times the previous number. Is there another description of the pattern that will give the same first three numbers but will lead to a different pattern? *Explain*.

31. CHALLENGE Consider the pattern $1, 1\dfrac{1}{2}, 1\dfrac{3}{4}, 1\dfrac{7}{8}, \ldots$.

a. *Describe* the pattern. Write the next three numbers in the pattern.

b. What is happening to the values of the numbers?

c. Make a conjecture about later numbers. *Explain* your reasoning.

○ = **WORKED-OUT SOLUTIONS** on p. WS1   ★ = **STANDARDIZED TEST PRACTICE**   ◆ = **MULTIPLE REPRESENTATIONS**

32. BASEBALL You are watching a pitcher who throws two types of pitches, a fastball (F, in white below) and a curveball (C, in red below). You notice that the order of pitches was F, C, F, F, C, C, F, F, F. Assuming that this pattern continues, predict the next five pitches.
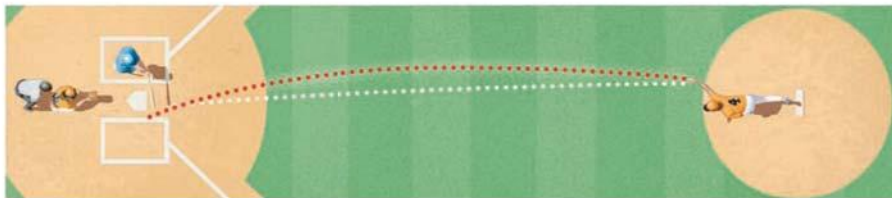
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33. STATISTICS The scatter plot shows the number of person-to-person e-mail messages sent each year. Make a conjecture that *could* be true. Give an explanation that supports your reasoning.
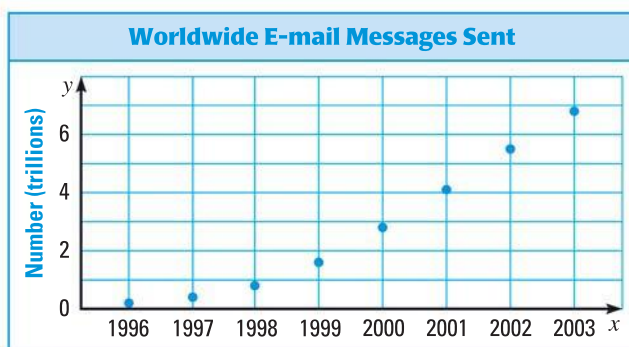
@HomeTutor for problem solving help at classzone.com

34. VISUAL REASONING Use the pattern below. Each figure is made of squares that are 1 unit by 1 unit.
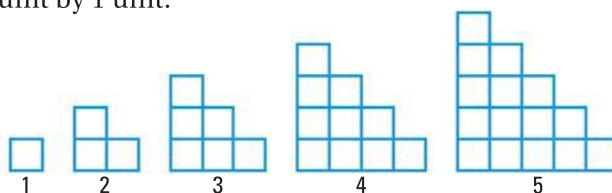
  a. Find the distance around each figure. Organize your results in a table.

  b. Use your table to *describe* a pattern in the distances.

  c. Predict the distance around the 20th figure in this pattern.

35. ◆ MULTIPLE REPRESENTATIONS Use the given function table relating *x* and *y*.

  a. Making a Table Copy and complete the table.

  b. Drawing a Graph Graph the table of values.

  c. Writing an Equation *Describe* the pattern in words and then write an equation relating *x* and *y*.

x	y
−3	−5
?	1
5	11
?	15
12	?
15	31

36. ★ **EXTENDED RESPONSE** Your class is selling raffle tickets for $.25 each.

   a. Make a table showing your income if you sold 0, 1, 2, 3, 4, 5, 10, or 20 raffle tickets.

   b. Graph your results. *Describe* any pattern you see.

   c. Write an equation for your income *y* if you sold *x* tickets.

   d. If your class paid $14 for the raffle prize, at least how many tickets does your class need to sell to make a profit? *Explain*.

   e. How many tickets does your class need to sell to make a profit of $50?

37. **FIBONACCI NUMBERS** The *Fibonacci numbers* are shown below. Use the Fibonacci numbers to answer the following questions.

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$$

   a. Copy and complete: After the first two numbers, each number is the __?__ of the __?__ previous numbers.

   b. Write the next three numbers in the pattern.

   c. **Research** This pattern has been used to describe the growth of the *nautilus shell*. Use an encyclopedia or the Internet to find another real-world example of this pattern.

38. **CHALLENGE** Set A consists of all multiples of 5 greater than 10 and less than 100. Set B consists of all multiples of 8 greater than 16 and less than 100. Show that each conjecture is false by finding a counterexample.

   a. Any number in set A is also in set B.

   b. Any number less than 100 is either in set A or in set B.

   c. No number is in both set A and set B.

MIXED REVIEW

Use the Distributive Property to write the expression without parentheses. *(p. 872)*

39. $4(x - 5)$ 40. $-2(x - 7)$ 41. $(-2n + 5)4$ 42. $x(x + 8)$

You ask your friends how many pets they have. The results are: 1, 5, 1, 0, 3, 6, 4, 2, 10, and 1. Use these data in Exercises 43–46. *(p. 887)*

43. Find the mean.   44. Find the median.   45. Find the mode(s).

46. Tell whether the *mean*, *median*, or *mode(s)* best represent(s) the data.

Find the perimeter and area of the figure. *(p. 49)*

47.

3 in.
7 in.

48.

4 cm

49.

10 ft
6 ft
8 ft